Exponential Growth and Decay

These notes are intended as a summary of section 5.3 (p. 358 – 363) in your workbook. You should also read the section for more complete explanations and additional examples.

Compound Interest

When money is invested in a savings account, it earns interest. If the interest is reinvested in the account, it also earns interest. This is called **compound interest**.

If a principal of P dollars is invested at an annual interest rate r (as a decimal), compounded n times per year, the amount, A, of the investment after t years is given by:

$$A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}$$

Example 3 (Sidebar p. 362)
A principal of \$1500 is invested at 4% annual interest, compounded quarterly. To the nearest quarter of a year, when will the amount be \$2500?

Exponential Growth / Decay

Compound interest is an example of **exponential growth**. Exponential growth occurs whenever the rate of growth is proportional to the function's current value. (i.e. the larger the value of your investment, the faster it earns interest)

In general, a function that models exponential growth will have the form:

$$y = a \cdot k^{bx} \qquad \begin{cases} k^b > 1 \\ a \in \mathbb{R} \\ b \in \mathbb{R} \\ k > 0 \end{cases}$$

where k is called the **growth factor**.

Exponential decay occurs whenever the rate of decrease is proportional to the function's current value. (i.e. as the value shrinks, the rate of decay slows)

In general, a function that models exponential decay will have the form:

$$y = a \cdot k^{bx} \qquad \begin{cases} 0 < k^b < 1 \\ a \in \mathbb{R} \\ b \in \mathbb{R} \\ k > 0 \end{cases}$$

where k is called the **decay factor**.

An example of exponential decay is the decrease in atmospheric pressure with altitude. At an altitude of h kilometers, the pressure P, in kilopascals, is modelled by the function:

$$P = 101.3 \cdot \left(0.88\right)^h$$

Example 4 (sidebar p. 363)
If the cabin pressure in an airplane is less than 70 kPa, passengers can suffer altitude sickness. To the nearest kilometer, at what altitude is the atmospheric pressure 70 kPa?

Homework: #12 – 14 in the exercises (p. 364 – 368). Answers on p. 369.